KANNUR UNIVERSITY
കണ്ണൂർ സർവകലാശാല
Re-accredited by NAAC with 'B++' Grade

Syllabus for Choice Based Credit and Semester System for Post Graduate Programme in Affiliated Colleges – 2023

MSc. MATHEMATICS

(2023Admission onwards)

KannurUniversity

Thavakkara,CivilStationP.O.KannurDistrict.

Kerala670002,India.

INDEX

PROGRAMME OUTCOMES

Program Outcomes (POs)

PO1. Advanced Knowledge and Skills: Postgraduate courses aim to provide students with in-depth knowledge and advanced skills related to their chosen field. The best outcome would be to acquire a comprehensive understanding of the subject matter and develop specialized expertise.

PO2. Research and Analytical Abilities: Postgraduate programs often emphasize research and analytical thinking. The ability to conduct independent research, analyze complex problems, and propose innovative solutions is highly valued.

PO3. Critical Thinking and Problem-Solving Skills: Developing critical thinking skills is crucial for postgraduate students. Being able to evaluate information critically, identify patterns, and solve problems creatively are important outcomes of these programs.

PO4. Effective Communication Skills: Strong communication skills, both written and verbal, are essential in various professional settings. Postgraduate programs should focus on enhancing communication abilities to effectively convey ideas, present research findings, and engage in academic discussions.

PO5. Ethical and Professional Standards: Graduates should uphold ethical and professional standards relevant to their field. Understanding and adhering to professional ethics and practices are important outcomes of postgraduate education.

PO6. Career Readiness: Postgraduate programs should equip students with the necessary skills and knowledge to succeed in their chosen careers. This includes practical skills, industry-specific knowledge, and an understanding of the job market and its requirements.

PO7. Networking and Collaboration: Building a professional network and collaborating with peers and experts in the field are valuable outcomes. These connections can lead to opportunities for research collaborations, internships, and employment prospects.

PO8. Lifelong Learning: Postgraduate education should instill a passion for lifelong learning. The ability to adapt to new developments in the field, pursue further education, and stay updated with emerging trends is a desirable outcome.

Programme Specific Outcomes (PSOs):

The M.Sc. Mathematics programme's main outcomes are

PSO1. Inculcate and develop mathematical aptitude and train students to apply their theoretical knowledge to solve problems

PSO2. Develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics

PSO3. Develop abstract, logical and critical thinking so that students can reflect critically upon their work and the work of others.

PSO4. Appreciate the international dimension of mathematics and its multicultural and historical perspectives.

PSO5. Develop in the student the ability to read, follow and appreciate mathematics.

PSO6. Train students to communicate mathematical ideas in a lucid and effectivemanner.

PSO7. Have a strong foundation in core areas of Mathematicsboth pure andapplied.

PSO8. Communicate mathematical ideas effectively, in writing as well asorally.

PSO9. Conduct Professional and Scholarly activities efficiently

PROGRAMME STRUCTURE

Duration: The duration of a Post graduate programme shall be four semesters inclusive of days of examinations distributed over a period of two academic years. The odd semesters (1, 3) shall be from June to October and the even semesters (2, 4) shall be from October/November to March. Each semester shall have 90 working days inclusive of days of all examinations. The minimum duration for completion of a two year Post graduate Programme in any subject is four semesters and the maximum period for completion is eight semesters from the date of registration.

Admission: Eligibility for admissions and reservation of seats for various First semester (Post Graduate) Programmes shall be according to the rules framed by the University from time to time. There shall be a uniform Academic cum Examinations calendar approved by the University for the registration, conduct and scheduling of examinations, and publication of results. The Academic cum Examinations Calendar shall be complied with by all colleges and offices, and the Vice Chancellor shall have all powers necessary for this purpose.

Courses: The Post graduate programme shall include three types of courses, viz., Core Courses, Elective Courses and Open Elective Courses (including MOOC courses) Parent Department shall offer appropriate elective courses for a specific programme. Open Elective Courses are offered either by the parent department or by any other Department or by via MOOC. Open Elective courses can be opted in third semester preferably having multidisciplinary in nature.

Project: There shall be a project work with dissertation to be undertaken by all students. Project and dissertation work is a special course involving application of knowledge in solving/analysing/exploring a real-life situation/problem. The dissertation entails field work, lab work, report, presentation and viva voce. Project with dissertation shall be done under the supervision of a faculty member of the department as per the curriculum design. A candidate may, however, in certain cases be permitted to work on the project in an industrial/ research organisation on the recommendation of the Head of the Department. In such cases, one of the teachers from the department concerned shall be the supervisor/internal guide and an expert from the industry/research organisation concerned shall act as co-supervisor/external guide. Projects shall be submitted in the last week of February in fourth semester. Belated and incomplete projects will not be entertained. Dissertation on project shall be prepared as per the guidelines given as Annexure 1.

Comprehensive Viva-Voce: There shall be a comprehensive viva-voce at the end of the programme covering questions from all courses of the programme.

Credits: Each course shall have a specified number of credits. The total credits required for successful completion of a four semester programme will be 80. Minimum credits for core course shall be 64. The number of credits from elective course/Open Elective course is 8 and for Dissertation and General Viva Voce , the maximum credits shall be 8.

Attendance: A student shall be permitted to appear for the semester examination, only if she/he secures not less than 75% attendance in all courses of a semester put together. Relaxation in this attendance shall be given according to the rules and regulations framed by the University from time to time..

Eligibility to register for examination: Only those students who are registered for the university examination with eligible attendance (including those under condonable limit) alone are eligible to be promoted to next semester. Students who have attendance in the prescribed limit but could not register for examination are eligible to move to the next semester after availing token registration. The candidates shall apply for token registration within two weeks of the commencement of the next semester. Token registration is allowed only once during the entire programme.

COURSE EVALUATION:

The evaluation scheme for each course shall contain two parts

a) Continuous Evaluation (CE)
b) End Semester Evaluation (ESE)

20% weightage shall be given to the Continuous Evaluation(CE) and 80% weightage shall be for the End Semester Evaluation (ESE)

Continuous Evaluation (CE):

20% of the total marks in each course are for continuous assessment. The continuous evaluation shall be based on a pre determined transparent system involving written test , assignments, seminar and Viva.

Sl. No.	Components	% of internal marks
1	Two test papers	40
2	Assignments	20
3	Seminar presentation of course study	20
4	Viva voce	20

Test Paper : For each course there shall be at least two class tests during a semester. Average mark of best two tests is to be considered for CE. The probable dates of the tests shall be announced at the beginning of each semester. Marks should be displayed on the notice board. Valued answer scripts shall be made available to the students for perusal within 10 working days from the date of the tests.

Assignment :- Each student shall be required to do two assignments for each course. Assignments after valuation must be returned to the students.

Seminar : Each student shall deliver one seminar as an internal component for every course and must be evaluated by the respective teacher in terms of structure.

Viva voce :-For each course there shall be a viva voce conducted by the faculty.

End Semester Evaluation (ESE)

End Semester Evaluation carries 80% of total marks. End Semester Evaluation of all semesters will be conducted in centralised valuation camps immediately after the examination.

Project Evaluation: Project evaluation shall be conducted at the end of fourth semester as per the following guidelines.

1. Evaluation of the Project Report shall be done under Mark System.
2. The evaluation of the project will be done at two stages:
   a) Continuous Evaluation(supervising teachers will assess the project and award internal marks)
   b) End Semester Evaluation(external examiner appointed by the University)
3. Marks secured for the project will be awarded to candidates, combining the Continuous Evaluation and End Semester Evaluation marks.
4. The Continuous Evaluation to End Semester Evaluation components is to be taken in the ratio 1:4. Assessment of different components may be taken as

below.

Continuous Evaluation(20% of total)	
Components	**Percentage**
Punctuality	10
Use of Data	20
Scheme/Organization of Report	40
Presentation and Viva voce	30

End Semester Evaluation (80% of total)	
Components	**Percentage**
Relevance and structure of the Topic	30
resentation of facts/figures/language style/diagrams etc	30
Findings and recommendations	10
Viva-Voce	30

Submission of the Project Report and presence of the student for viva are compulsory for Continuous Evaluation. No marks shall be awarded to a candidate if she/he fails to submit the Project Report for End Semester Evaluation. The student should get a minimum of 40% marks of the aggregate and 40% separately for ESE for pass in the project. There shall be no improvement chance for the Marks obtained in the Project Report. In an instance of inability of obtaining a minimum pass marks as required, the project work shall be re- done and the report may be re-submitted along with subsequent exams through parent department.

Project evaluation and viva-voce shall be conducted by at least two external examiners. 15 minutes time shall be given for the presentation and another15 minutes for viva voce for each candidates. Maximum number of candidates for evaluation shall not be more than 8 per day.

Comprehensive Viva-Voce: There shall be a comprehensive viva voce at the end of the programme covering questions from all courses of the programme. The viva voce shall be conducted by two external examiners. 30 minutes shall be given for each candidates. Maximum number of candidates for this also shall not be more than 8 per day.

Structure of Question paper for ESE (80 marks)

PART	No of Questions in the QP	No of Questions to be answered	Marks of each question	Total Marks	%
A	6	5	4	20	25%
B	5	3	7	21	26.25%
C	5	3	13	39	48.75%

Total Number of questions = 6+5+5 = 16

Number of questions to be answered = 5 + 3 + 3=11

Total Marks in the question paper =6x4 + 5x7+ 5x13= 24+ 35+65= 124

Level of questions based on revised Bloom's taxonomy

PART	Revised Bloom's taxonomy	
	Level	
A	1, 2	Remembering, Understanding
B	6	Creating
C	3, 4, 5	Applying, Analysing, Evaluating

Time distribution for answering each unit

PART	No of Questions in the QP	No of Questions to be answered	Approximate time to answer a question (minutes)	Total Time (minutes)
A	6	5	8	40
B	5	3	20	60
C	5	3	25	75

GRADING:

Indirect grading system shall be adopted for the assessment of a student's performance in a course(both CE and ESE). Each course is evaluated by assigning marks with a letter grade (A^{+}, A, B, C, D, E and F) to that course by the method of indirect grading. Mark system is followed instead of direct grading for each question. For each course in the semester, letter grade, grade point and percentage of marks are introduced in the indirect grading system with scale as per guidelines given below:

% of Marks (CE+ESE)	**Grade**	**Interpretation**
90 and above	A+	Outstanding
80 to below90	A	Excellent
70 to below80	B	Very Good
60 to below 70	C	Good
50 to below 60	D	Satisfactory
40 to below50	E	Pass
Below 40	F	Failure

Evaluation (both CE and ESE)is carried out using Mark system .The grading on the basis of a total CE and ESE marks will be indicated for each course. Each letter grade is assigned a '**Grade point'** (GP) which is a point obtained using the formula:

Grade Point = (Total marks awarded / Total Maximum marks) x 10.

'**Credit point'** (CP) of a course is the value obtained by multiplying the grade point (GP) by the credit (C) of the course:

$$CP = GP \times C$$

A minimum of grade point 4 is needed for the successful completion of a course.A candidate securing not less than 40% of aggregate marks of a course with not less than 40% in End Semester Evaluation(ESE)and not less than 10% in Continuous Evaluation (CE) separately shall be declared to have passed in that course. A minimum of grade point 4 with letter grade E is needed for the successful completion of a course.Appearance for Continuous evaluation (CE) and End Semester Evaluation (ESE) are compulsory and no grade shall be awarded to a candidate if she/he is absent for CE/ESE or both. After the successful completion of a semester, Semester Grade Point Average (SGPA) of a student in that semester is calculated using the formula given below.

SGPA = Sum of the Credit Points of all courses in a semester / Total Credits in that semester

Semester Grade Point Average' (SGPA) is the value obtained by dividing the sum of credit points obtained by a student in the various courses taken in a semester by the total number of credits in that semester. SGPA determines the overall performance of a student at the end of a semester. For the successful completion of a semester, a student should pass all courses in that semester. However, a student is permitted to move to the next semester irrespective of SGPA obtained.SGPA shall be rounded off to three decimal places.

The **Cumulative Grade Point Average** (CGPA) of the student is calculated at the end of each semester. The CGPA of a student determines the overall academic level of the student in each stage of the programme. CGPA can be calculated by the following formula:

CGPA = Sum of Credit Points of all completed semesters / Total Credits acquired

CGPA shall be rounded off to three decimal places.

At the end of the programme, the over all performance of a candidate is indicated by the Overall Grade Point Average . Overall **Grade Point Average (OGPA)** of the student is calculated at the end of the programme. The OGPA of a student determines the overall academic level the student in a programme and is the criterion for classification and ranking the students. OGPA can be calculated by the following formula

GPA = Sum of Credit Points obtained in all semesters of the programme / Total Credits (80)

OGPA shall be rounded off to three decimal places.

An overall letter grade for **OGPA** for the entire programme shall be awarded to a student after completing the entire programme successfully .Over all letter grade based on OGPA and conversion of Grades into classification shall be in the following way.

Grade range OGPA	Overall Letter Grade	Classification
9 - 10	A+	First class with Distinction
8 - 8.999	A	
7 - 7.999	B	First class
6 - 6.999	C	
5 - 5.999	D	Second class
4 - 4.999	E	Pass
Below 4	F	Fail

The Percentage of marks based on OGPA is calculated by multiplying them by 10.

Percentage in two decimal places = [OGPA in three decimal places] x 10%

A student who fails to secure a minimum mark for a pass in a course is permitted to write the examination along with the subsequent batch.

SEMESTER WISE DETAILS:

SEMESTER –I					
Number of Core Courses: 5					
Sl. No	**Course Code**	**Course Title**	**Lecture Hours/Week**	**Marks (Internal+External)**	**Credits**
1	MSMAT01C01	Abstract Algebra	5	100	4
2	MSMAT01C02	Linear Algebra	5	100	4
3	MSMAT01C03	Real Analysis	5	100	4
4	MSMAT01C04	Topology	5	100	4
5	MSMAT01C05	Ordinary Differential Equations	5	100	4
	Total credit in core courses		**25**	**500**	**20**
	Total credits in Semester - I				20

SEMESTER –II					
Number of Core Courses: 5					
Sl. No	**Course Code**	**Course Title**	**Lecture Hours/Week**	**Marks (Internal+External)**	**Credits**
1	MSMAT02C01	Advanced Abstract Algebra	5	100	4
2	MSMAT02C02	Measure Theory	5	100	4
3	MSMAT02C03	Advanced Real Analysis	5	100	4
4	MSMAT02C04	Advanced Topology	5	100	4
5	MSMAT02C05	PDE and Integral Equations	5	100	4
	Total credit in core courses		**25**	**500**	**20**
	Total credits in Semester - II				20

SEMESTER –III					
Number of Core Courses: 4					
Sl. No	**Course Code**	**Course Title**	**Lecture Hours/Week**	**Marks (Internal+External)**	**Credits**
1	MSMAT03C01	Functional Analysis	5	100	4
2	MSMAT03C02	Complex Analysis	5	100	4
3	MSMAT03C03	Number Theory	5	100	4
4	MSMAT03C04	Differential Geometry	5	100	4
	Total credit in core courses		**20**	**400**	**16**
Number of Open Elective Courses: 1					
5	MSMAT03O01/02/ 03/04/05/06/07	Open Elective Course 1	5	100	4
	Total credit in open elective courses		**5**	**100**	**4**
	Total credits in Semester - III				**20**

SEMESTER –IV					
Number of Core Courses: 2					
Sl. No	**Course Code**	**Course Title**	**Lecture Hours/Week**	**Marks (Internal+External)**	**Credits**
1	MSMAT04C01	Operator Theory	5	100	4
2	MSMAT04C02	Complex Function Theory	5	100	4
	Total credit in core courses		**10**	**200**	**8**
Number of Core Elective Courses: 1					
3	MSMAT04E01/ 02/03/04/05/06	Elective Course 1	5	100	4
	Total credit in core elective courses		**5**	**100**	**4**
4	MSMAT04P01: Project/Dissertation		**10**	**100**	**4**
5	MSMAT04V01: Comprehensive Viva - Voce		**--**	**100**	**4**
	Total credits in Semester - IV				**20**

ELECTIVE COURSES:

Open Elective Courses: III Semester	
COURSE CODE	COURSE TITLE
MSMAT03O01	Graph Theory
MSMAT03O02	Discrete Mathematics
MSMAT03O03	Operations Research
MSMAT03O04	Calculus of Variations
MSMAT03O05	Fuzzy Mathematics
MSMAT03O06	Coding Theory
MSMAT03O07	Automata and Formal Languages

Core Elective Courses : IV Semester	
COURSE CODE	COURSE TITLE
MSMAT04E01	Commutative Algebra
MSMAT04E02	Fourier and Wavelet Analysis
MSMAT04E03	Probability Theory
MSMAT04E04	Algebraic Topology
MSMAT04E05	Numerical Analysis and Computing
MSMAT04E06	Advanced Differential Geometry

<u>SEMESTER I</u>

MSMAT01C01: Abstract Algebra

Course Objective: To provide a first approach to the subject of algebra, which is one of the basic pillars of modern mathematics and to gain knowledge in basic group theory and ring theory which are essential for further study.

Course Learning outcome: After successful completion of the course, student will be able to understand the basic algebraic structures such as group theory and ring theory.

Unit I

Direct Products and finitely generated Abelian Groups, Group Action on a Set
(Chapter 2: Section 9; Chapter 3: Section 14)

Unit II

SylowTheorems, Applications of Sylow Theorems, Field of Quotients of the Integral Domain.
(Chapter 4: Section 17; Chapter 6: Section 26)

Unit III

Isomorphism Theorems, Series of Groups, Free Abelian Groups
(Chapter 4: Section 16, 18, 19)

Unit IV

Ring of Polynomials, Factorization of Polynomials over a Field, Homomorphisms and Factor Rings, Prime and Maximal Ideals
(Chapter 6: Section 27, 28, 30, 31)

Text Book: John. B. Fraleigh, Neal E. Brand, *A First Course in Abstract Algebra* (Eighth Edition)**,** Pearson, 2021

References:

1. Joseph A. Gallian, *Contemporary Abstract Algebra*, Narosa, 1999
2. I. N. Herstein, *Topics in Algebra*, Wiley India Pvt. Ltd, 2006
3. M. Artin, *Algebra* (Second Edition) Addison Wesley, 2010
4. David S. Dummit, *Abstract Algebra* (Third Edition), Wiley India, 2011
5. D. S. Malik, John. N. Merdson, M. K. Sen, *Fundamentals of Abstract Algebra*, McGraw-hill Publishing Co., 1996
6. Allan Clark, *Elements of Abstract Algebra*, Dover Publications, 1984
7. David M. Burton, *A First course in Rings and Ideals*, Addison-Wesley Educational Publishers, 1970

MSMAT01C02: Linear Algebra

Course Objective: To understand the basic ideas of Vector spaces, Linear transformations, Decomposition of Linear operators and Inner product spaces which are essential to learn advanced level mathematics.

Course Learning outcome: After successful completion of the course, student will be able to understand the basic linear algebra- vector space, linear transformations and inner product spaces.

Unit I

Linear Transformations: Linear Transformations, The algebra of Linear Transformations, Isomorphism, Representation of Transformation by Matrices
(Chapter 3: Sections 3.1 to 3.4,)

Unit II

Linear Functionals, The Double Dual, The Transpose of a Linear Transformation.
Elementary Canonical Forms: Introductions, Characteristic values
(Chapter 3: Sections 3.5 to 3.7 Chapter 6: Section 6.1 and 6.2)

Unit III

Elementary Canonical Forms: Annihilating Polynomials, Invariant Subspace, Simultaneous Triangulations& Simultaneous Diagonalisation, Direct Sum Decompositions, Invariant Direct Sums
(Chapter 6: Sections 6.3 to 6.7)

Unit IV

Elementary Canonical Forms: The Primary Decomposition Theorem.
Cyclic Subspaces and Annihilators, Cyclic Decomposition and Generalized Cayley Hamilton Theorem (Proof of Cyclic Decomposition Theorem is excluded)
Inner Product Spaces: Inner Products, Inner Product Spaces,
(Chapter 6: Section 6.8; Chapter 7: Sections: 7.1, 7.2 (upto Theorem 4), Chapter 8: Sections 8.1, 8.2)

Text Book: Kenneth Hoffman & Ray Kunze: *Linear Algebra* (Second Edition), Prentice- Hall of India Pvt. Ltd, 2015.

References:

1. Stephen H Friedberg, Arnold J Insel and Lawrence E Spence, *Linear Algebra* (Fourth Edition), Prentice Hall, 2015.
2. Sheldon Axler*, Linear Algebra Done Right* (Third Edition), Springer, 2015
3. Martin Anthony and Michele Harvey, *Linear Algebra: Concepts and Methods,*Cambridge University Press, 2012
4. S. Kumaresan, *Linear Algebra: A Geometric Approach*, PHI Learning Pvt. Ltd., 2000
5. Serge A Lang, *Linear Algebra* (Third Edition), Springer, 2004.
6. Paul R Halmos, *Finite-Dimensional Vector Spaces*, Springer 1974.
7. Michael Artin, *Algebra* (Second Edition) Addison Wesley, 2010

MSMAT01C03: Real Analysis

Course objective: To develop basic concepts like limit, convergence, differentiation and Riemann integral. Convergence of functions.

Course Learning outcome: After successful completion of the course, student will be able to understand the basic real analysis- convergence, differentiation and integration.

Unit I

Basic Topology-Finite, Countable and uncountable Sets, Metric spaces, Compact Sets, Perfect Sets, Connected Sets. (Chapter 2)

Unit II

Continuity-Limits of function, Continuous functions, Continuity and compactness, continuity and connectedness, Discontinuities, Monotonic functions, Infinite limits and Limits at infinity.
(Chapter 4)

Unit III

Differentiation, Derivative of a real function. Mean value theorems, Continuity of derivatives. L Hospital's rule. Derivatives of higher order. Taylor's theorem. Differentiation of vector valued functions. (Chapter 5)

Unit IV

Reimann – Stieltjes integral. Definition and existence of the integral. Integration and differentiation. Integration of vector – valued functions. Rectifiable curves. (Chapter 6)

Text Book: Walter Rudin, *Principles of Mathematical Analysis* (Third Edition), McGraw Hill, 1976.

References:

1. T. M. Apostol, *Mathematical Analysis* (Second Edition), Narosa, 2002
2. R. G. Bartle, *The Elements of Real Analysis*(Second Edition), Wiley International, 1975
3. G. F. Simmons*, Introduction to Topology and Modern Analysis*, McGraw Hill, 2017
4. Charles Chapman Pugh,*Real Mathematical Analysis*, Springer, 2010
5. Sudhir R. Ghorpadeand Balmohan V. Limaye, *A Course in Calculus and Real Analysis*, Springer, 2006
6. R. G. Bartle and D. R Sherbert, *Introduction to Real Analysis*, John Wiley Bros., 1982
7. L. M Graves, *The Theory of Functions of a Real Variable*, Tata McGraw- Hill, 1978
8. M. H Protter and C.B Moray, *A First course in Real Analysis*, Springer, 1977

MSMAT01C04: Topology

Course Objective: To present an introduction to the theory of topology, a powerful tool for understanding other branches of mathematics.

Course Learning outcome: After successful completion of the course, student will be able to understand the topological spaces, continuous functions and connected spaces.

Unit I

Topological spaces, Basis for a topology, The order topology, The product topology (finite).
(Chapter 2: Sections 12 to 15)

Unit II

The subspace topology, Closed sets and limit points, Continuous functions.
(Chapter 2: Sections 16 to 18)

Unit III

The product topology, The metric topology (Theorem 20.4 & Theorem 20.5 statement only), The metric topology (continued), The Quotient Topology.
(Chapter 2: Sections 19 to 22)

Unit IV

Connected spaces, Connected subspace of the real line, Components and local connectedness.
(Chapter 3: Sections 23 to 25)

Text Book: James. R. Munkres, *Topology- A First Course*, Pearson India (Second Edition), 2014.

References:

1. C. Wayne Patty, *Foundations of Topology*, Joes and Bartlett, 2010.
2. K. Parthasarathy, *Topology - An invitation*, Springer2022
3. G.F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 2017.
4. K. D. Joshi, *Introduction to General Topology*, New age International, 1983
5. Singer and J. A. Thorpe, *Lecture Notes on Elementary Topology and Geometry*, Springer, 1967.
6. J. L. Kelley, *General Topology*, Springer, 1975.
7. Stephen Willard, *General Topology*, Dover Books, 1970.

MSMAT01C05: Ordinary Differential Equations

Course Objective: To gain knowledge on the basic differential equations at the heart of analysis which is a dominant branch of mathematics for 300 years. This subject is the natural purpose of the primary calculus and the most important part of mathematics for understanding physics.

Course Learning outcome: After successful completion of the course, student will be able to understand the basics of differential equations and the method of solving them.

Unit I

Introduction: A Review of Power Series, Series Solutions of First Order Equations, Second Order Linear Equations. Ordinary Points, Regular Singular Points, Regular Singular Points (Continued), Gauss's Hyper Geometric Equation, The Point at Infinity. (Chapter 5: Sections 26 to 32)

Unit II

Legendre Polynomials, Properties of Legendre Polynomials, Bessel Functions, The Gamma Function, Properties of Bessel functions (Chapter 8: Sections 44 to 47)

Unit III

Oscillations and the Sturm Separation Theorem, The Sturm Comparison Theorem, General Remarks on Systems, Linear Systems, Homogeneous Linear Systems with Constant Coefficients.
(Chapter 4: Sections 24 and 25; Chapter 10: Sections 54 to 56)

Unit IV

The Method of Successive Approximations, Picard's Theorem, Systems, The Second Order Linear Equation
(Chapter 13: Sections 69 to 71)

Text Book: G.F Simmons, *Differential Equations with Historical Notes* (Third Edition), CRC Press-Taylor and Francis Group, 2017.

References:

1. G. Birkoff and G. C Rota, *Ordinary Differential Equations* (Fourth Edition), Wiley and Sons, 1978.
2. E. A Coddington, *An Introduction to Ordinary Differential Equations*, Prentice Hall of India, 1974.
3. P. Hartmon, *Ordinary Differential Equations*, Society for Industrial and aplied.., 1987
4. Chakraborti, *Elements of Ordinary Differential Equations and Special Functions*, Wiley Eastern, 1990
5. L.S Poutrigardian, *A Course in Ordinary Differential Equations*, Hindustan Publishing Corp., 1967
6. S.G Deo and V.Raghavendra, *Ordinary Differential Equations and Stability Theory*, Tata McGraw Hill, 1967
8. V. I. Arnold, *Ordinary Differential Equations*, MIT Press Cambridge, 1981
9. A. K. Nandakumaran, P. S. Dutty and Raju K George, *Ordinary Differential Equations: Principles and Applications*, Cambridge University Press, 2017

SEMESTER II

MSMAT02C01: Advanced Abstract Algebra

Course Objective: The aim of this course is to learn advances in algebra and Galois Theory.

Course Learning outcome: After successful completion of the course, student will be able to understand some topics in algebra, including Galois theory.

Unit I

Unique Factorization Domains, Euclidean Domains, Gaussian Integers and Multiplicative Norms
(Chapter 7: Sections 34 to 36)

Unit II

Introduction to Extension Fields, Algebraic Extensions, Geometric Constructions, Finite Fields
(Chapter 8: Sections 39 to 42)

Unit III

Introduction to Galois Theory , Splitting Fields , The Isomorphism Extension Theorem
(Chapter 9: Sections 43 and 44).

Unit IV

Separable Extensions. Galois Theory (Chapter 9: Sections 45 and 46).

Text Book: John. B. Fraleigh, Neal E. Brand, *A First Course in Abstract Algebra* (Eighth Edition), Pearson, 2021

References:

1. Joseph A. Gallian, *Contemporary Abstract Algebra*, Narosa, 1999
2. I. N. Herstein, *Topics in Algebra*, Wiley India Pvt. Ltd, 2006
3. M. Artin, *Algebra* (Second Edition) Addison Wesley, 2010
4. David S. Dummit, *Abstract Algebra* (Third Edition), Wiley India, 2011
5. D. S. Malik, John. N. Merdson, M. K. Sen, *Fundamentals of Abstract Algebra*, McGraw-hill Publishing Co., 1996
6. Allan Clark, *Elements of Abstract Algebra*, Dover Publications, 1984
7. David M. Burton, *A First course in Rings and Ideals*, Addison-Wesley Educational Publishers, 1970

MSMAT02C02: Measure Theory

Course Objective: The aim of this course is to learn the Measure Theory, Lebesgue Integration and related Concepts

Course Learning outcome: After successful completion of the course, student will be able to understand some topics in measure theory, Lebesgue integration.

Unit I

Measure on the real line: Lebesgue Outer measure, Measurable sets, Regularity, Measurable Functions, Borel and Lebesgue Measurability (Including Theorem 17),
(Chapter 2: Sections 2.1 to 2.5).

Unit II

Integration of functions of a Real Variable: Integration of Non-negative Functions, Integration of functions of a Real Variable, The general Integral, Riemann and Lebesgue Integrals.
(Chapter 3: Sections 3.1, 3.2 and 3.4)

Unit III

Abstract Measure Space: Measures and Outer measures, extension of measure, Uniqueness of the extension, Measure Spaces
(Chapter 5: Sections 5.1 to 5.3, 5.5)

Unit IV

Abstract Measure Space: Integration with respect to a Measure,**Inequalities and the L^p Spaces:** The L^p Spaces, The inequalities of Holder and Minkowski, Completeness of $L^p(\mu)$
(Chapter 5; Section 5.6; Chapter 6: Sections 6.1, 6.4 and 6.5)

Text Book: G De Barra, *Measure Theory and Integration*. (Second Edition), New Age International Pvt. Ltd., 2003.

Reference:

1. Walter Rudin, *Real and Complex Analysis* (Third Edition), Tata McGraw Hill, 2017
2. H. L Royden, P M Fitzpatrick, *Real Analysis*, Pearson, Fourth Edition, Pearson, 2015
3. R. G. Bartle, *The Elements of Integration and Measure Theory*, John Wiley and Sons, 1995
4. P.R Halmos, *Measure Theory,* Springer, 1976
5. A. E Taylor, *General Theory of Functions and Integrations*, Dover Publications, 2010
6. Inder K. Rana, *An Introduction to Measure and Integration*, Narosa Publishing House, 1997
7. M.Thamban Nair,*Measure and Integration: A First Course*, CRC Press, 2019

MSMAT02C03: Advanced Real Analysis

Course Objective: The aim of this course is to learn the real analysis in advanced level. The course provides the basis for further studies within functional analysis, topology and function theory.

Course Learning outcome: After successful completion of the course, student will be able to understand uniform convergence and functions of several variables.

Unit I

Sequence and series of Functions: Discussion of Main Problem, Uniform Convergence, Uniform Convergence and Continuity. (Chapter 7: Sections 7.1 to 7.15)

Unit II

Uniform Convergence and Integration, Uniform Convergence and Differentiation, Equicontinous Families of Functions, The Stone-Weierstrass Theorem (Chapter 7: Sections 7.16 to 7.33)

Unit III

Some Special Functions: Power Series, The Exponential and Logarithmic Functions, The Trigonometric Functions, The Algebraic Completeness of the Complex Field, Fourier Series, The Gamma Function (Chapter- 8)

Unit IV

Functions of Several Variables: Norm and invertible linear Operators, Differentiation, The Contraction Principle, The Inverse Function Theorem, The Implicit Function Theorem (Chapter 9: Sections 9.6 to 9.29)

Text Book: Walter Rudin, *Principles of Mathematical Analysis* (Third Edition) McGraw Hill, 1986.

Reference:

1. T.M. Apostol, *Mathematical Analysis* (Second Edition), Narosa, 2002
2. R. G. Bartle, *The Elements of Real Analysis*(Second Edition), Wiley International, 1975
3. G.F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 2017
4. Charles Chapman Pugh,*Real Mathematical Analysis*, Springer, 2010
5. S. R. Ghorpadeand B. V. Limaye, *A Course in Calculus and Real Analysis*, Springer, 2006
6. R. G Bartle and D. R Sherbert, *Introduction to Real Analysis*, John Wiley Bros., 1982
7. L. M Graves, *The Theory of Functions of a Real Variable*, Tata McGraw Hill, 1978
8. M. H Protter and C.B Moray, *A First course in Real Analysis*, Springer, 1977

MSMAT02C04: Advanced Topology

Course Objective: To learn advanced level theory of topology, a powerful tool for understanding other branches of mathematics.

Course Learning outcome: After successful completion of the course, student will be able to understand Compactnes, Separation Axioms and classical Theorems in topology such as Urysohn Lemma, UrysohnMetrization theorem, Tietze Extension, Tychonoff Theorem and Stone – CechCompactification

Unit I

Well ordered sets, Compact spaces, Compact subset of the real line, Limit point compactness, Local compactness (Chapter 1: Section 10, Chapter 3: Sections 26, 27, 28 and 29)

Unit II

The Countability Axioms, The separation Axioms, Normal Spaces. (Chapter 4: Section 30, 31, 32)

Unit III

The Urysohn Lemma, The UrysohnMetrization theorem, The Tietze Extension Theorem (Chapter 4: Section 33, 34 , 35)

Unit IV

The Tychonoff Theorem, The Stone – CechCompactification (Chapter 5: Section 37, 38)

Text Book: James. R. Munkres, *Topology- A First Course*, Pearson India (Second Edition), 2015.

References:

1. C. Wayne Patty, *Foundations of Topology*, Joes and Bartlett, 2010.
2. K. D. Joshi, *Introduction to General Topology*, New age International, 1983
3. G.F. Simmons*, Introduction to Topology and Modern Analysis*, McGraw Hill, 2017.
4. Singer and J. A. Thorpe, *Lecture Notes on Elementary Topology and Geometry*, Springer, 1967.
5. J. L. Kelley, *General Topology*, Springer, 1975.
6. Stephen Willard, *General Topology*, Dover Books, 1970.

MSMAT02C05: PDE and Integral Equations

Course Objective: To learn about partial differential equations, integral equations and finding solutions using various techniques

Course Outcome: Upon the successful completion of the course students will learn techniques to solve first order PDE and analyse the solution to get information about the parameters involved in the model and get an idea about Integral equations

Unit I

First-order equations: Introduction, Quasilinear equations, The method of characteristics, Examples of the characteristics method, The existence and uniqueness theorem, The Lagrange method, General nonlinear equations, Exercises.
(Chapter 2: Sections 2.1 to 2.6, 2.9,2.10 from Text 1)

Unit II

Second-order linear equations in two independent variables: , Classification, Canonical form of hyperbolic equations, Canonical form of parabolic equations, Canonical form of elliptic equations
The one-dimensional wave equation: Introduction, Canonical form and general solution, The Cauchy problem and d'Alemberts formula, Domain of dependence and region of influence, The Cauchy problem for the nonhomogeneous wave equation
(Chapters 3 and 4 from Text 1)

Unit III

The method of separation of variables: Introduction, Heat equation: homogeneous boundary condition, Separation of variables for the wave equation, Separation of variables for nonhomogeneous equations, The energy method and uniqueness, Further applications of the heat equation.
Elliptic equations: Introduction, Basic properties of elliptic problems, The maximum principle, Applications of the maximum principle, Greens identities, The maximum principle for the heat equation, Separation of variables for elliptic problems, Poissons formula.
(Chapters 5 and 7 from Text 1)

Unit IV

Integral Equations: Introduction, Relations between differential and integral equations, The Green's functions, Fredholom equations with separable kernels, Illustrative examples, Hilbert- Schmidt Theory, Iterative methods for solving Equations of the second kind. The Newmann Series, Fredholm Theory [Chapter 3:Sections 3.1 to 3.3, 3.6 to 3.11 from the Text 2]

Text Books:
1. YehudhaPinchover and Jacob Rubienstein, *An Introduction to Partial Differential Equations*, Camridge University Press, 2005
2. F. BHildebrand, *Methods of Applied Mathematics*(Second Edition), Prentice Hall, 1972.

References:

1. Amaranath T, *Partial Differential Equations*, Narosa, 1997.
2. A. Chakrabarti, *Elements of ordinary Differential Equations and special functions*, Wiley Eastern Ltd, 1990
3. E.A. Coddington, *An Introduction to Ordinary Differential Equtions*, Printice Hall of India, 1974
4. R. Courant and D. Hilbert, *Methods of Mathematical Physics*-Vol I, Wiley Eastern Reprint, 1975

SEMESTER III

MSMAT03C01: Functional Analysis

Course Objective: To enable the students to connect topology, linear algebra and analysis together to get into the theory of functional analysis.

Course Learning outcome: After successful completion of the course, student will be able to bring together the theories of linear algebra, topology and analysis and get acquainted with the basic theories of functional analysis.

Unit I

Complete Metric spaces: Definition of Metric space ,Sequence space l^{∞}, Function space C[a,b]. Examples of Metric Spaces : Sequence space s.Space B(A) of bounded functions. Space l^{p}, Hilbert sequence space l^{2}, Hölder and Minkowski inequalities for sums. Definition of Dense set and separable space. Non separability of space l^{∞} and separability of Space l^{p} , 1≦p<+∞. Definition of Cauchy sequence and completeness. Completeness of l^{∞}. Completeness of c, Completeness of l^{p} , 1≦p<+∞. Completeness of space C[a,b]. (All Proofs of Statements , Results and Examples of these topics from Chapter 1 is excluded)

Normed space and Banach space: Normed space and Banach space, Further Properties of Normed spaces, Finite dimensional normed spaces and subspaces, compactness and finite dimension.
(Chapter 1, Section 1.1-1, 1.1-6, 1.1-7, 1.2-1, 1.2-2, 1.2-3, 1.3-5, 1.3-9, 1.3-10, 1.4-3, 1.5-2, 1.5-3,1.5-4, 1.5-5 Statements only, Proof excluded).
(Chapter 2: Sections 2.2 to 2.5)

Unit II

Linear operators and linear functionals : Linear operators, Bounded and continuous linear operators, linear functionals. Linear operators and functionals on finite dimensional spaces. Normed Spaces of Operators, Dual Space.
(Chapter 2: Section 2.6 to 2.10)

Unit III

Inner Product spaces and Hilbert spaces: Inner Product space. Hilbert space. Properties of Inner Product Spaces, Orthogonal Complements and Direct Sums, Orthonormal Sets and Sequences, Series Related to Orthonormal Sequences and Sets. Total Orthonormal Sets and Sequences.
(Chapter 3: Section 3.1 to 3.6).

Unit IV

Functionals and Operators on Hilbert spaces: Representation of Functionals on Hilbert spaces, Hilbert-Adjoint operator, Self adjoint, unitary and normal operators.
Fundamental Theorems for Normed and Banach Spaces: Zorn's lemma, Hahn-Banach theorem, Hahn-Banach theorem for complex vector spaces and normed spaces.
(Chapter 3: Section 3.8 to 3.10; Chapter 4: Section 4.1 to 4.3).

Text Book: Erwin Kreyszig,*Introductory Functional Analysis with Applications*, John Wiley,1991
References:

1. Balmohan V Limaye*, Functional Analysis* (Second Edition), New Age International Publishers, 1996
2. M.T Nair, *Functional Analysis: A First Course*; Prentice Hall of India, 2021
3. Walter Rudin, *Functional Analysis*,McGraw-Hill Inc.,US, 1978
4. B.Choudhary, Sudarsan Nanda, *Functional Analysis with Applications* (Second edition), New Age International (P) Ltd., 2003
5. Walter Rudin, *Real and Complex Analysis*, McGraw Hill Education, 2017
6. J.B Conway, *A Course in Functional Analysis*, Springer, 2010

MSMAT03C02: Complex Analysis

Course Objective: To enable the students to develop basic complex analysis techniques for advanced level complex functiona theory

Course Learning outcome: After successful completion of the course, student will study Cauchys theorems, residue integration and space of analytic and meromorphic functions.

Unit I

Power Series representation of analytic functions, Zeros of an analytic function, The index of a closed curve, Cauchy's theorem and Integral formula, The homotopic version of Cauchy's Theorem and simple connectivity
(Chapter 4: Sections 2 to 6)

Unit II

Counting zeros, the Open Mapping Theorem, Goursat's Theorem. Classification of Singularities, Residues
(Chapter 4: Sections 7 and 8; Chapter 5: Sections 1 and 2)

Unit III

The Argument Principle, The Maximum Principle, Schwarz's Lemma
(Chapter 5: Section 3; Chapter 6: Sections 1 and 2)

Unit IV

The space of continuous functions $C(G,\Omega)$(Omit proof of Arzela-Ascoli Theorem), Spaces of analytic functions, The Reimann Mapping Theorem
(Chapter 7: Sections 1, 2 and 4 (Omit Theorem 1.23))

Text Book: John B Conway, *Functions of One Complex Variable* (Second Edition), Springer, 1995

Reference:

1. Louis Pennise, *Elements of Complex Variable*, Half, Richart& Winston, 1976
2. H. Silverman, *Complex Variable*, Haughton Miffin Complex, Boston, 1975.
3. Walter Rudin, *Real and Complex Analysis* (Third Edition) McGraw Hill International Edition, 1967
4. E. T Copson, *An Introduction to the Theory of a Complex Variables*, Oxford University Press, 1974.
5. Lars V. Ahlfors, *Complex Analysis* (Third Edition), McGraw Hill Education, 2017
6. Theodore W. Gamelin, *Complex Analysis*, Springer, 2001

MSMAT03C03: Number Theory

Course Objective: To enable the students to develop basic concepts in Number Theory.

Course Learning outcome: After successful completion of the course, student will study the basics of both Analytic and Algebraic Number Theory.

Unit I

Arithmetical Functions and Dirichlet multiplication: Introduction- The Mobius function $\mu(n)$– The Euler totient function ϕ (n) –The relation connecting μ and ϕ -the product formula for ϕ (n) – The Dirichlet product of arithmetical functions- Dirichlet inverses and Mobius inversion formula- The Mangolt function $\Lambda(n)$ –Multiplicative functions- Multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function- Liouville's function $\lambda(n)$- The divisor function $\sigma_\alpha(n)$.

(Text 1: Section 2.1 to 2.13)

Congruences: Residue classes and complete residue system- Liner Congruences-Reduced residue system and the Euler- Fermat theorem- Polynomial congruences modulo $p$and Langrange's theorem- Applications of Langrange's theorem- Simultaneous linear congruences and Chinese Remainder theorem- Applications of Chinese remainder theorem- Polynomial congruences with prime power moduli.

(Text 1: Section 5.2 to 5.9)

Unit II

Quadratic Residues and Quadratic Reciprocity Law: Quadratic residues- Legendre's symbol and its properties- Evaluation of $(-1 | p)$ and $(2 | p)$ Gauss lemma-The quadratic reciprocity law – Applications of the reciprocity law – The Jacobi symbol- Applications to Diophantine equations.

(Text 1: Sections 9.1 to 9.8)

Primitive Roots: The exponent of number mod m and primitive roots- Primitive roots and reduced residue; system- The nonexistence of primitive roots mod 2a for $\alpha \geq 3$- The existence of primitive roots mod p for odd primes p- Primitive roots and quadratic residues – The existence of primitive roots and P^a- The existence of primitive roots mod 2 P^a –The nonexistence of Primitive roots in the remaining cases- The number of primitive roots mod m.

(Text 1: Sections 10.1 to10.9)

Unit III

Algebraic Backgrounds: Symmetric polynomials- modules- free abelian groups
(Text 2, Section 1.4-1.6)
Algebraic Numbers: Algebraic numbers- Conjugates and Discriminants- Algebraic integers
(Text 2: Section 2.1 to 2.3)

Unit IV

Algebraic Numbers: Integral bases- Norms and Traces- Rings of integers.
(Text 2: Section 2.4 to 2.6)
Quadratic and Cyclotomic fields: Quadratic fields-Cyclotomic fields.
(Text 2: Sections 3.1 to 3.2)

Text Book:

1. Tom M Apostol,*Introduction to Analytic Number Theory*, Springer, 1976
2. lan Stewart and David Tall,*Algebraic Number Theory and Fermat's last theorem* (Third Edition), A K Peters/CRC Press, 2001

References

1. D. M. Burton,*Elementary Number Theory* (Sixth Edition),McGraw Hill, 2005
2. G. H. Hardy and E. M. Wright,*Introduction to the theory of numbers*, Oxford International Edn, 1985
3. Hurwitz & N. Kritiko,*Lectures on Number Theory*, Springer Verlag, 1986
4. T. Koshy, Elementary Number Theory with Applications; Harcourt / Academic Press, 2002
5. D. Redmond,*Number Theory: Monographs & Texts in Mathematics No: 220*, Marcel DekkerInc., 1994
6. P. Ribenboim,*The little book of Big Primes*, Springer Verlag, New York, 1991
7. K.H. Rosen,*Elementary Number Theory and its applications* (Third Edn.), Addison Wesley PubCo., 1993
8. P. Samuel,*Theory of Algebraic Numbers*, Herman Paris Houghton Mi in, NY, 1975
9. S. Lang, *Algebraic Number Theory*, Addison Wesley Pub Co., Reading, Mass, 1970)
10. Z.I. Borevich& I.R.Shafarevich, *Number Theory*, Academic Press, NY, 1966.
11. Esmonde and Ram Murthy,*Problems in Algebraic Number Theory*, Springer Verlag, 2000.

MSMAT03C04: Differential Geometry

Course Objective: The course gives an introduction to the elementary concepts of differential geometry using the calculus of vector fields so that the students also attain a deep understanding of several variables calculus.

Course Learning outcome: After successful completion of the course, student will be able to understand the basics of differential geometry and several variable calculus.

Unit I

Graphs and Level Sets, Vector Fields, The Tangent Space, Surfaces, Vector Fields on Surfaces; Orientation

(Chapters 1 to 5)

Unit II

The Gauss Map, Geodesics, Parallel Transport
(Chapters 6 to 8)

Unit III

Weingarten Map, Curvature of Plane Curves Arc Length and Line Integrals,

(Chapters 9 to 11)

Unit IV

Curvature of Surfaces, Parametrized Surfaces.

(Chapters 12 and 14)

Text Book: J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer- Verlag, 1979.

References:

1. Andrew Priestley, *ElementaryDifferential Geometry*, Springer, 2001
2. Sean Dineen, *Multivariate Calculus and Geometry*, Springer, 1998
3. D. J. Struik, *Classical Differential Geometry* (Second Edition), Dover, 1988
4. E. Kreyszig, *Introduction to Differential Geometry and Riemannian Geometry*, Univ. of Toronto Press, 1969
5. Manfredo P. Do Carmo, *Differential Geometry of Curves and Surfaces*(Second Edition), Dover Books, 2016
6. Barrot O Neill, *Elementary Differential Geometry*(Revised Second Edition), Academic Press, 1997

SEMESTER IV

MSMAT04C01: Operator Theory

Course Objective: To enable the students to advance from functional analysis to operator theory

Course Learning outcome: After successful completion of the course, student will be able to understand the advanced level operator theory and their interplay with other branches of higher mathematics.

Unit I

Fundamental Theorems for Normed and Banach Spaces: Adjoint operator, Reflexive spaces (Definitions and statement of Results from section 4.6), Definition of Category, Baire's Category Theorem (Statement only proof excluded), Uniform Boundedness Theorem. Definition of Strong convergence. Definition of Weak convergence. Open Mapping Theorem, Closed Linear Operators, Closed Graph Theorem.
(Chapter 4: Sections 4.5, 4.6(Definitions and statement of Results only), 4.7-1, 4.7-2 (Statement only), 4.7-3, 4.8-1, , 4.8-2, 4.12 and 4.13).

Unit II

Approximation Theory: Approximation in Normed Spaces, Uniqueness, Strict Convexity, Approximation in Hilbert Space.
Spectral Theory in Normed Spaces: Basic Concepts, Spectral Properties of Bounded Linear Operators. Further Properties of Resolvent and Spectrum, Use of Complex Analysis in Spectral Theory.
(Chapter 6: Sections 6.1, 6.2 and 6.5; Chapter 7: Sections 7.2 to 7.5)

Unit III

Compact Operators and Their Spectrum: Compact Linear Operators on Normed Spaces, Further Properties of Compact Linear Operators.Spectral Properties of Compact Linear Operators on Normed Spaces, Further Spectral Properties of Compact Linear Operators.
(Chapter 8: Sections 8.1 to8.4)

Unit IV

Spectral Theory of Bounded Self-Adjoint Linear Operators: Spectral Properties of Bounded Self-Adjoint Linear Operators, Further Spectral Properties of Bounded Self-Adjoint Linear Operators, Positive Operators,Square Roots of a Positive Operator(Statement only proof of the theorem is excluded) , Projection Operators, Further Properties of Projections.
(Chapter 9: Sections 9.1 to 9.6 (All proofs in section 9.4 are omitted).

Text Text Book: Erwin Kreyszig,*Introductory Functional Analysis with Applications*, John Wiley,1991

References:

1. Balmohan V Limaye*, Functional Analysis* (Second Edition), New Age International Publishers, 1996
2. M.T Nair, *Functional Analysis: A First Course*; Prentice Hall of India, 2021
3. Walter Rudin, *Functional Analysis*,McGraw-Hill Inc.,US, 1978
4. B. Choudhary, Sudarsan Nanda, *Functional Analysis with Applications* (Second edition), New Age International (P) Ltd., 2003
5. Walter Rudin, *Real and Complex Analysis*, McGraw Hill Education, 2017
6. J.B Conway, *A Course in Functional Analysis*, Springer, 2010

MSMAT04C02: Complex Function Theory

Course Objective: To enable the students to acquire advanced level knowledge in complex function theory

Course Learning outcome: After successful completion of the course, student will develop knowledge in advanced complex analysis and would be capable to apply these knowledge in solving Harmonic PDEs.

Unit I

Weierstrass Factorization Theorem, Factorization of the sine function, The gamma function (Chapter 7: Sections 4 to 7)

Unit II

The Riemann zeta function, Runge's Theorem, Simple connectedness
(Chapter 7: Section 8; Chapter 8: Sections 1 and 2))

Unit III

Mittag–Leffler's Theorem, Schwarz Reflection Principle, Analytic continuation along a path
(Chapter 8: Section 3; Chapter 9: Sections 1 and 2)

Unit IV

Mondromy Theorem, Basic Properties of Harmonic functions, Harmonic functions on a disk
(Chapter 9: Section 3; Chapter 10: Sections 1 and 2))

Text Book: John B Conway, *Functions of One Complex Variable* (Second Edition), Springer, 1995

References:

1. Louis Pennise, *Elements of Complex Variable*, Half, Richart& Winston, 1976
2. H. Silverman, *Complex Variable*, Haughton Miffin Complex, Boston, 1975.
3. Walter Rudin, *Real and Complex Analysis* (Third Edition) McGraw Hill International Edition, 1967
4. E. T Copson, *An Introduction to the Theory of a Complex Variables*, Oxford University Press, 1974.
5. Lars V. Ahlfors, *Complex Analysis* (Third Edition), McGraw Hill Education, 2017
6. Theodore W. Gamelin, *Complex Analysis*, Springer, 2001

MSMAT04P01: Project/Dissertation

Course Objective: Enable students to learn independently and orient them towards research in Mathematics.

Course Learning Outcome: After successful completion of the project work, student will be able to study or research in a topic that is beyond the regular classroom learning in both rigor and content. Further, students will be able to produce reports that exhibit both the background and the conclusions reached as a result of such study or research.

Guidelines for project:

1. The project work can be done in any advanced level topic of Mathematics.
2. The student should not have studied this topic in the regular class.
3. Each student must be assigned a supervisor from the Department for carrying out the project work.
4. Students may present the progress of the project work to the supervisor regularly, which shall be considered for the continuous evaluation of the project by the supervisor
5. The project report must be self-contained as far as possible
6. The students must submit a report at the end of the project, which is duly signed by the supervisor on or before the date stipulated by the Department.

Annexure 1;

Guidelines for the preparation of dissertation on project:

1. Arrangement of contents shall be as follows:
   1.Cover page and title page
   2. Bonafide certificate
   3. Declaration by the student
   4. Acknowledgement
   5. Table of contents
   6. List of tables
   7. List of figures
   8. List of symbols, Abbreviations and Nomenclature
   9. Chapters
   10. Appendices
   11. References

2. Page dimension and typing instruction:
   The dimension of the dissertation on project should be in A4 size. The dissertation should be typed in bond paper and bound using flexible cover of the thick white art paper or spiral binding. The general text shall be typed preferably in Latex with font size 12. The content of the report shall be around 40 pages.

3. Bonafide certificate shall be in the following format:

CERTIFICATE

This is to certify that the project entitled ....(title) submitted to the Kannur University in partial fulfilment of the requirements of Post Graduate Degree in ....(subject), is a bonafide record of studies and work carried out by ....(Name of the student) under my supervision and guidance.

Office seal Signature, name, designation and official address of the Supervisor.
Date

4. Declaration by the student shall be in the following format:

DECLARATION

I...(Name of the candidate) hereby declare that this project titled ....(title) is a bonafide record of studies and work carried out by me under the supervision of ....(Name, designation and official address of the supervisor), and that no part of this project, except the materials gathered from scholarly writings, has been presented earlier for the award of any degree or diploma or other similar title or recognition.

Date: Signature and name of the student

MSMAT04V01: Comprehensive Viva - Voce

Course Objective: The objective of comprehensive viva - voce is to ensure the overall knowledge and effective communication of each student in various topics of the syllabus.

Course Learning Outcome: After the successful completion of the comprehensive viva-voce, students will have an overall idea about the various topics covered in the course. Students will also be able to face interviews with confidence.

Guidelines for comprehensive viva-voce:

1. Questions can be from any topic studied in the entire course with more emphasis on core papers.
2. Students may be asked to present any topic decided by examiners.

Elective Papers:

Open Elective Papers (Third Semester)

MSMAT03O01 : Graph Theory

Course Objective: To enable the students to acquire advanced level knowledge in Graph Theory

Course Learning outcome: After successful completion of the course, student will develop knowledge n connectivity in graphs, Independent sets and Matchings, Edge and vertex colourings and related concepts.

Unit I

Graphs and Subgraphs: Graphs and Simple graphs, Graph Isomorphism, Incidence and adjacency matrix, Subgraphs, Vertex degrees, Paths and Connection, Cycles.
Trees: Trees, Cut Edges and Bonds, Cut Vertices.
(Chapter 1: Sections 1.1 to 1.7; Chapter 2: Sections 2.1 to 2.3)

Unit II

Connectivity: Connectivity, blocks.
Euler Tours and Hamilton Cycles: Euler Tours, Hamilton Cycles(up to (including) Lemma 4.4.1)
Matchings: Matchings, Matchings and Coverings in bipartite Graphs.
(Chapter 3: Section 3.1 and 3.2; Chapter 4: Sections 4.1 and 4.2 (Excluding Lemma 4.4.2 to Corollary 4.6 and related Exercises);Chapter 5: Sections 5.1 and 5.2)

Unit III

Edge Colourings: Edge Chromatic Number, Vizing's Theorem.
Vertex Colorings: Chromatic Number, Book's Theorem, Hajo's Conjecture.
(Chapter 6: Sections 6.1 and 6.2; Chapter 8: Sections 8.1 to 8.3)

Unit IV

Independent Sets and Cliques: Independent Sets, Ramsey's Theorem.
Planar Graphs: Plane and Planar Graphs, Dual Graphs, Euler's Formula, Bridges
(Chapter 7: Section 7.1 and 7.2; Chapter 9: Section 9.1 to 9.4)

Text Book: J.A.Bondy and U.S.Murty, *Graph Theory with Applications*, The MacMillan Press Ltd, 1976

References:

1. F. Harrary, *Graph Theory*, Narosa Publishing House, 2001
2. NarasinghDeo, *Graph Theory with applications to Engineering andComputer Science*, PHI,1979
3. O. Ore, *Graph and Their uses*, Random House Inc, NY, 1963
4. K. D. Joshi, *Foundations of Discrete Mathematics*, Wiley Eastern Ltd, 1989

MSMAT03O02: Discrete Mathematics

Course Objective: To enable the students to acquire advanced level knowledge in Discrete Mathematics

Course Learning outcome: After successful completion of the course, student will develop knowledge in Combinatorics and Graph theory

Unit I

General Counting Methods for Arrangement and Selections, Generating Functions

(Chapter 5: Sections 5.1 to 5.5; Chapter 6: Sections 6.1 to 6.5)

Unit II

Recurrence Relations and Inclusion and Exclusion

(Chapter 7: Sections 7.1 to 7.5; Chapter 8: Sections 8.1 to 8.3)

Unit III

Polya's Enumeration Formula

(Chapter 9: Sections 9.1 to 9.4)

Unit IV

Quick Review of Basic Concepts in Graph Theory as in Chapter 1 of the text

(Questions from this chapter need not be included for the external examination)

Search Trees and Spanning Trees, The Travelling Salesperson Problem, Tree Analysis of Salesperson Problems, Tree Analysis of Sorting Algorithms

(Chapter 3: Sections 3.2 to 3.4)

Text: Alan Tucker, *Advanced Combinatorics*(Sixth Edition), John Wiley and Sons Inc., 2012

References:

1. R. Brualdi, *Introductory Combinatorics*, Prentice Hall Inc., 5th Edition, 2009.
2. D. Mazur, *Combinatorics: A guided tour*, Mathematical Association of America, 2009.
3. R. Merris, *Combinatorics* (Second edition) John Wiley and Sons, 2003.
4. W. Wallis and J. George, *Introduction to Combinatorics*, CRC Press, 2011.

MSMAT03O03: Operations Research

Course Objective : Identify and develop the mathematical tools that are needed to solve optimization problems.

Course Learning outcome: After successful completion of the course, student will be able to understand different techniques involved in operations research.

Unit I

Linear programming in two-dimensional spaces. General LP problem. Feasible, basic and optimal solutions, simplex method, simplex tableau, finding the first basic feasible solution, degeneracy, simplex multipliers. (Chapter 3: Sections 1 to 15).

Unit II

The revised simplex method. Duality in LP problems, Duality theorems, Applications of duality, Dual simplex method, summary of simplex methods, Applications of LP.
(Chapter 3: Sections 16 to 22)

Unit III

Transportation and Assignment problems
(Chapter 4)

Unit IV

Integer programming. Theory of games
(Chapters 6 and 12)

Text Book: K. V. Mital and C. Mohan, *Optimisation Methods in Operations Research and Systems Analysis* (Third Edition), New Age International, 1996

References:

1. Wagner, *Operations Research*, Prentice Hall India
2. A. Ravindran, Don T. Philips, James Solberg, *Operations Research, Principles and Practice* (Third Edition), John Wiley
3. G. Hadley, *Linear Programming*, Addison Wesley
4. KantiSwarup, P.K.gupta, Man Mohan, *Operations Research*, S. Chand & Co., 2007
5. Premkumar Gupta and D. S. Hira, *Operations Research*, S. Chand & Company Ltd., 1995.

MSMAT03O04: Calculus of Variations

Course Objective: To enable the students to acquire basic knowledge in Calculus of Variations

Course Learning outcome: After successful completion of the course, student will be able to understand the basic theory of calculus of variations, get acquainted with Euler equations and apply them in solving extremal problems.

Unit I

Elements of the Theory: Functionals- Some simple variational problems, Function spaces, The variation of a functional- A necessary condition for an extremum, The simplest variational problem- Euler's equation, The case of several variables, A simple variable end point problem, The variational Derivative, Invariance of Euler's equation,
Further Generalizations: The fixed end point problem for *n* unknown functions
(Chapter 1; Chapter 2: Section 9)

Unit II

Further Generalizations: Variational problems in parametric form, Functional Depending on higher order derivatives, Variational Problems with subsidiary conditions
General Variations of a Functional: Derivation of the basic formula, End points lying on two given curves or surfaces,
(Chapter 2: Sections 10 to 12; Chapter 3: Sections 13 to 14)

Unit III

General Variations of a Functional: Broken extremals –The Weierstrass-Erdmann conditions
The Canonical Form of the Euler Equations and related topics: The canonical form of the Euler equations, First Integrals of the Euler equations, The Legendre transformation, Canonical transformations, Noether's Theorem, The principle of least action, Conservation laws, The Hamilton-Jacobi Equation- Jacobi's Theorem
(Chapter 3: Section 15; Chapter 4)

Unit IV

The Second Variation- Sufficient condition for a Weak Extremum: Quadratic functionals- The second variation of a functional, The formula for the second variation- Legendre's condition, Analysis of the quadratic functional $\int_a^b (Ph'^2 + Qh^2)dx$, Jacobi's necessary condition- More on conjugate points, Sufficient conditions for a weak extremum
(Chapter 5: Sections 24 to 28)

Text Book:I M. Gelfand and S.V Fomin, *Calculus of Variations*, Prentice Hall Inc, N.Y, 1963
References:
1. G. A. Bliss, *Calculus of Variations*, Open Court Publishing Co. Chicago, 1925
2. O. Bolza, *Lecture on Calculus of Variations*, G.E Stinchar & Co. NY, 1931
3. R. Courant and D. Hilbert,*Methods of Mathematical Physics* (Vol. 1), Wiley Eastern, 1975
4. I. Elsgoltz, *Differential Equations and Calculus of Variations*, Mr Publishers Moscow, 1973
5. M. Morse, *The Calculus of Variations*, American Mathematical Society, 1934

MSMAT03O05: Fuzzy Mathematics

Course Objective: The aim is to provide an introduction to the fundamental concepts of Fuzzy Mathematics.

Course Learning outcome: After successful completion of the course, student will be able to understand the basics of fuzzy mathematics.

Unit I

From classical (crisp) sets to fuzzy sets: characteristics and significance of the paradigm shift. Additional properties of α-cuts. Representation of fuzzy sets. Extension principle for fuzzy sets.
(Chapter 1 & Chapter 2)

Unit II

Operations on fuzzy sets. Types of operations. Fuzzy complements. t-norms, t- conorms. Combinations of operations. Aggregate operations , Fuzzy numbers Arithmetic operations on intervals. Arithmetic operations on fuzzy numbers. Lattice of fuzzy numbers
(Chapter 3: Sections 3.1 to 3.4; Chapter 4:Sections 4.1 to 4.5)

Unit III

Crisp and fuzzy relations, projections and cylindric extensions, binary fuzzy relations, binary relations on a single set, Fuzzy equivalence relations , Compatibility and ordering relations.
(Chapter 5:Sections 5.1 to5.6)

Unit IV

Fuzzy morphisms. sup-i, inf- ωicompositions of fuzzy relations. Fuzzy logic. Fuzzy propositions. Fuzzy quantifiers. Linguistic hedges. Inference from conditional, conditional and qualified and quantified propositions
(Chapter 5: Sections 5.8 to 5.10 ; Chapter 8)

Text Book:G. J. Klir and Bo Yuan ,*Fuzzy sets and Fuzzy logic Theory and Applications,* PHI, 1995

References:

1. H. J. Zimmermann, *Fuzzy Set Theory and its Applications*, Kluwer, 1985
2. H. J. Zimmermann, *Fuzzy Sets, Decision Making and Expert Systems*, Kluwer, 1987
3. D. Dubois & H. Prade,*Fuzzy Sets and Systems: Theory and Applica*tions, Academic Press, 1980

MSMAT03O06: Coding Theory

Course Objective: The aim is to introduce the fundamental concepts of coding theory.

Course Learning outcome: After successful completion of the course, student will be able to understand the basics of coding theory.

Unit I

Introduction to Coding Theory. Correcting and detecting error patterns. Weight and distance. MLD and its reliability. Error-detecting codes. Error correcting codes. Linear codes
(Chapter 1; Chapter 2: Sections 2.1 to 2.5)

Unit II

Generating matrices and encoding. Parity check matrices. Equivalent codes. MLD for linear codes. Reliability of IMLD for linear codes , Some bounds for codes,Perfect codes , Hamming codes. Extended codes extended Golay code and Decoding of extended Golay code
(Chapter 2: Sections 2.6 to 2.12; Chapter 3: Sections3.1 to 3.6)

Unit III

The Golay code, Reed-Muller codes, Fast decoding of RM(1,m) , Cyclic linear

codes. Generating and parity check matrices for cyclic codes. Finding cyclic codes. Dual cyclic codes (Chapter 3: Sections 3.7 to 3.9; Chapter 4)

.Unit IV

BCH codes. Decoding 2-error-correcting BCH code. Reed-Solomon codes.Decoding
(Chapter 5; Chapter 6: Sections 6.1 to 6.3)

Text Book : – D.R. Hankerson, D. G. Hoffman, D. A. Leonard, C. C. Lindner, K. T. Phelps, C. A. Rodger and J. R. Wall , *Coding Theory and Cryptography The Essentials* (Second edition) , Marcel Dekker (2000)

References:

1.J. H. van Lint,*Introduction to Coding Theory*, Springer Verlag, 1982
2.E. R. Berlekamp,*Algebraic Coding Theory,* McGraw Hill, 1968

MSMAT03O07: Automata and Formal Languages

Course Objective: To enable the students to acquire basic knowledge in Automata Theory

Course Learning outcome: After successful completion of the course, student will be able to understand the basic theory of Automata and Formal languages.

Unit I

Strings, Alphabets and Languages, Finite state systems, Basic definitions, Nondeterministic finite automata, Finite automata with ϵ moves.
(Chapter 1:Section 1.1; Chapter 2: Sections 2.1 to 2.5)

Unit II

Regular expressions, Finite automata and regular expressions, Finite automata Applications of regular expressions, The pumping lemma for regular languages, Closure properties of regular sets, Decision properties for regular sets, Minimizing finite automata.
(Chapter 3: Sections 3.1 to 3.3; Chapter 4: Sections 4.1 to 4.4)

Unit III

Motivation and introduction to CFG, Context-free grammars, Derivation trees, Ambiguity in context-free grammars, Informal description, Definitions of Pushdown Automata and Context free languages, The language of a PDA, Equivalence of PDA's and CFG's
(Chapter 5: Sections 5.1, 5.2, 5.4; Chapter 6: Sections 6.1 to 6.3) (Proof of Theorem 6.9 excluded)

Unit IV

Context free languages, pumping lemma for context free languages, Closure properties of context free languages, Introduction to Turing machine, The Turning Machine model, Programming techniques for Turing Machines
(Chapter 7: Sections 7.1 to 7.3; Chapter 8: Sections 8.1 to 8.3)

Text Book: John E. Hopcroft, Rajeev Motwani and Jeffery D. Ullman, *Introduction to Automata Theory, Languagesand Computation* (Second Edition), Pearson, 2001.

References:

1. G.E.Revesz, *Introduction to Formal Languages*, Dover, 2012.
2. K. L. P. Mishra, N. Chandrasekharan, *Theory of Computer Science: Automata Languages and Computation*, PHI, 2006.
3. M. Sipser, *Introduction to the Theory of Computation*, (Third edition), Cengage India Private Limited, 2014.
4. P.Linz, *An Introduction to Formal Languages and Automata* (Sixth Edition), Jones and Bartlettudent,, 2012.

Core Elective Papers - Fourth Semester

MSMAT04E01: Commutative Algebra

Course Objective: To enable the students to acquire basic knowledge in commutative algebra

Course Learning outcome: After successful completion of the course, student will be able to understand the basic theory of commutative algebra and get acquainted with ideals, modules and decomposition theorems.

Unit I

Rings and Ideals: Rings and Ring Homomorphism, Ideals, Quotient Rings, Zero Divisors, Nilpotent Elements, Unit, Prime Ideals and Maximal Ideals, Nilradical and JocobsonRedical, Operations on Ideals, Extension and Contraction
(Chapter 1)

Unit II

Modules: Modules and Module Homomorphism, Submodules and Quotient Modules, Operations on Sub modules, Direct Sum and Product, Finitely Generated Modules, Exact Sequences.
Rings and Modules of Fractions: Rings and Modules of Fractions, Local Properties,
(Chapter 2 (relevant sections); Chapter 3(relevant sections))

Unit III

Rings and Modules of Fractions: Extended and Contracted Ideals
Primary Decomposition: Primary Decomposition.
(Chapter 4; Chapter 5 (relevant sections))

Unit IV

Chain Conditions: Chain conditions
Noetherian Rings: Primary decomposition in Noetherian Rings
(Chapter 6; Chapter 7 (Omit Proposition 7.8, Proposition 7.9 and Corollary 7.10)

Text Book: M. F. Atiyah and I. G. Macdonald, *Introduction to communicative Algebra*, Addison Wiley, 1969

References:

1. N. Bourbaki, *Commutative Algebra*, Paris Herman, 1961
2. D. Burton, *A first course introduction to Rings and Ideals*, Wesley, 1970
3. N. S. Gopalakrishnan, *Commutative Algebra*, Oxonian Press, 1984
4. T. W. Hunger ford, *Algebra*, Springer Verlag, 1974
5. D. G. North Cott, *Ideal Theory*, Cambridge University Press, 1953
6. O. Zarisiki and P. Samuel, *Commutative Algebra* (Vol I and II). Van Nostrand, Princeton, 1960

MSMAT04E02: Fourier and Wavelet Analysis

Course Objective: To enable the students to acquire basic theory of Fourier and wavelet analysis

Course Learning outcome: After successful completion of the course, student will be able to understand the basic theory of wavelet analysis in Z_n, Z and R.

Unit I

Wavelets on $\boldsymbol{Z_n}$: Construction of Wavelets on Z_n- The First Stage, Construction of Wavelets on Z_n- The Iteration Step.
(Chapter 3: Sections 3.1 and 3.2)

Unit II

Wavelets on $\boldsymbol{Z_n}$: The Haar System, the Shannon wavelets and the Daubechies's D6 wavelets on Z_n.
Wavelets on $\boldsymbol{Z}$: $l^2(Z)$, Complete Orthonormal sets in Hilbert Spaces, $L^2([-\pi, \pi])$ and Fourier Series,
(Chapter 3: Examples 3.32, 3.33 and 3.35 of Section 3.3.; Chapter 4: Sections 4.1 to 4.3)

Unit III

Wavelets on $\boldsymbol{Z}$: The Fourier transforms and convolution on $l^2(Z)$, First Stage Wavelets on Z, The Iteration Step for Wavelets on Z.
(Chapter 4: Sections 4.4 to 4.6)

Unit IV

Wavelets on $\boldsymbol{R}$:$L^2(R)$ and Approximate Identities, The Fourier Transform on R.
(Chapter 5: Section 5.1 to 5.2)

Text Book: M.W. Frazier, *An Introduction to Wavelets through Linear Algebra*, Springer, 1999

References:

1.G. Bachman, L. Narici, E. Beckenstein, *Fourier and Wavelet Analysis*, Springer, 2000
2. I. Daubechies, *Ten Lectures on Wavelets*, SIAM, 1992
3. C. Heil, *A Basis Theory Primer*, Birkhauser, 2011
4. D. F. Walnut, *An Introduction to Wavelet Analysis*, Birkhauser, 2002

MSMAT04E03: Probability Theory

Course Objective: To enable the students to acquire advanced level knowledge in probability theory.

Course Learning Outcome: After successful completion of the course, student will develop knowledge in defining the probability space of a random variable, the distribution function and convergence of distribution function of a random variable.

Unit I

Sets and Classes of Events, Random Variables
(Chapter 1: sections 1.1 to 1.4; Chapter 2: Sections 2.1 to2.3)

Unit II

Probability Spaces, Distribution Functions.
(Chapter 3: Sections 3.1 to 3.5; Chapter 4: Sections 4.1 to 4.4)

Unit III

Expectation and Moments, Convergence of Random Variables.
(Chapter 5: Section 5.1 to 5.3; Chapter 6: Sections 6.1 to 6.6)

Unit IV

Characteristic Functions, Convergence of Distribution Functions.
(Chapter 7: Section 7.1 to 7.5; Chapter 8: Sections 81 to 8.3)

Text Book: B.R Bhat, *Modern Probability Theory* (Second Edition), New Age International Pvt Ltd, New Delhi.

References:

1. A.K. Basu, *Measure and Probability*, Prentice Hall of India, 2003.
2. A.N Kolmogrov, *Foundations of Probability*, Chelsea, NY, 1950
3. M. Loeve, *Probability Theory*, Van-Nostrand, Princeton, 1963
4. Y. S. Chows and H. Tiecher, *Probability Theory*, Springer Verlag, 1988
5. P. Billingsley,*Probability and Measure*, John Wiley & Sons, NY(1979)

MSMAT04E04: Algebraic Topology

Course Objective: To enable the students to acquire advanced level knowledge in Algebraic Topology.

Course Learning Outcome: After successful completion of the course, student will be able to understand the basics of algebraic topology and understand the fundamental group from a different perspective.

Unit I

Geometric Complexes and Polyhedra: Introduction, Examples, Geometric Complexes and Polyhedra, Orientation of Geometric Complexes.
Chapter 1: Sections 1.1 to 1.4)

Unit II

Simplicial Homology Groups: Chains, Cycles, Boundaries and Homology Groups, Examples of Homology Groups, The Structure of Homology Groups, The Euler-Poincare Theorem, Pseudomanifolds, and Homology Groups of S^n.
(Chapter 2: Sections 2.1 to 2.5)

Unit III

Simplicial Approximation: Introduction, Simplicial Approximation, Induced Homomorphisms on the Homology Groups, TheBrouwer Fixed Point Theorem and Related Results.
(Chapter 3: Sections 3.1 to 3.4)

Unit IV

The Fundamental Group: Introduction, Homotopic Paths and The Fundamental Group, The Covering Homotopy property for S^1, Examples of Fundamental Groups.
(Chapter 4: Sections 4.1to 4.4)

Text Book: Fred H. Croom, *Basic Concepts of Algebraic Topology*, Springer, 1978

References

1. S. Eilenberg and N. Steenrod,*Foundations of Algebraic Topology*, Princeton univ. Press; 1952
2. S. T. Hu,*Homology Theory*, Holden-Day, San Farancisco, 1966
3. C. T. C. Wall,*A Geometric Introduction to Topology*, Addison-Wesley Pub. Co. Reading Mass, 1972
4. W. S. Massey, *Algebraic Topology: An Introduction*, Springer Verlag N Y, 1977

MSMAT04E05: Numerical Analysis and Computing

Course Objective: The aim is to provide an introduction to the fundamental concepts of numerical analysis and computing.

Course Learning Outcome: After successful completion of the course, student will be able to understand different methods of finding numerical solutions of a system of equations.

Unit I

Principles of Numerical Calculations : Common Ideas and Concepts, Fixed-Point Iteration, Newton's Method, Linearization and Extrapolation, Finite Difference Approximations, Some Numerical Algorithms, Solving a Quadratic Equation, Recurrence Relations. Divide and Conquer Strategy. Matrix Computations, Matrix Multiplication, Solving Linear Systems by LU Factorization, Sparse Matrices and Iterative Methods, Software for Matrix Computations. The Linear Least Squares Problem, Basic Concepts in Probability and Statistics, Characterization of Least Squares Solutions, The Singular Value Decomposition, The Numerical Rank of a Matrix. Numerical Solution of Differential Equations, Euler's Method , Introductory Example, Second Order Accurate Methods.
(Chapter 1: Sections 1.1 to 1.5 except 1.5.4)

Unit II

How to Obtain and Estimate Accuracy, Basic Concepts in Error Estimation, Sources of Error, Absolute and Relative Errors, Rounding and Chopping, Computer Number Systems, The Position System, Fixed- and Floating-Point Representation, IEEE Floating-Point Standard, Elementary Functions, Multiple Precision, Accuracy and Rounding Errors, Floating-Point Arithmetic, Basic Rounding Error Results, Statistical Models for Rounding Errors, Avoiding Overflow and Cancellation.
(Chapter 2: Sections 2.1 to 2.3)

Unit III

Interpolation and Approximation: The Interpolation Problem, Bases for Polynomial Interpolation, Conditioning of Polynomial, Interpolation Formulas and Algorithms, Newton's Interpolation ,Inverse Interpolation, Barycentric Lagrange Interpolation, Iterative Linear Interpolation, Fast Algorithms for Vandermonde Systems, The Runge Phenomenon, Generalizations and Applications, Hermite Interpolation, Complex Analysis in Polynomial Interpolation, Rational Interpolation, Multidimensional Interpolation, Piecewise Polynomial Interpolation, Bernštein Polynomials and Bézier Curves, Spline Functions, The B-Spline Basis, Least Squares Splines Approximation. The Fast Fourier Transform. The FFT Algorithm.
(Chapter 4: Sections 4.1 to 4.4 and 4.7.1)

Unit IV

Numerical Integration: Interpolatory Quadrature Rules ,Treating Singularities, Classical Formulas, Super-convergence of the Trapezoidal Rule, Higher-Order Newton–Cotes' Formulas, Integration by Extrapolation , The Euler–Maclaurin Formula, Romberg's Method, Oscillating Integrands Adaptive Quadrature
(Chapter 5: Sections 5.1 except 5.1.6,5.2)

Text Book: GermundDahlquist, AkeBjorck,*Numerical Analysis in Scientific Computing*(Vol.1), Cambridge University press, 2008

References:

1. William H. Press, *Numerical Recipes in C – The art of scientific computing* (Third edition), Cambridge University Press, 1992
2. Laurene V. Fausett, *Applied Numerical Analysis using MATLAB*. (Second edition), Pearson, 2007
3. David Kincaid, Chency et.al., *Numerical Analysis: Mathematics of Scientific Computing,*(Third edition), Cengage Learning (Pub), 2012
4. Deuflhard P. & A. Hofmann,*Numerical Analysis in Modern Scientific Computing*, Springer, 2002.

MSMAT04E06: Advanced Differential Geometry

Course Objective: The aim is to introduce the advanced level topics of differential geometry

Course Learning Outcome: After successful completion of the course, student will be able to understand andapply the theory ofmanifolds and Lie algebra

Unit I

Differential Manifolds, Smooth maps and diffeomorphisms,, Tangent spaces to a manifold, Derivatives of smooth maps,Immersions and submersions, Submanifolds.

(Chapter 2: Sections 2.1 to 2.6)

Unit II

Vector fields, Flows and exponential map, Frobenius theorem, Lie groups and Lie algebras, Homogenous spaces

(Chapter 2: Sections 2.7 to 2.11)

Unit III

Tensor Analysis, Multilinear algebra, Exterior algebra, Tensor fields, The exterior derivative, Lie derivatives

(Chapter 3: Sections 3.1 to 3.5)

Unit IV

Integration, Orientable manifolds, Integration on manifolds,Stoke's theorem

(Chapter 4: Sections 4.1 to 4.3)

Textbook: S. Kumaresan, *A Course in Differential Geometry and Lie groups*, Hindustan Book Agency, 2002

References:

1. John M. Lee*, Introduction to smooth manifolds*, Springer, 2002
2. M.Spivak, *A Comprehensive introduction to Differential Geometry*, Vol 1., Publish or perish 1970
3. Loring. W. Tu. , *An Introduction to manifolds*, Springer, 2011
4. J. M. Lee, *Manifolds and Differential Geometry*, American Mathematical Society, 2009
5. J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer- Verlag, 1979

Kannur University

Model Question Paper

FIRST SEMESTER M.Sc. DEGREE EXAMINATION

Mathematics

MSMAT01C01: Abstract Algebra

CBCSS - 2023 Admission Onwards

Time: Three Hours **Maximum : 80 Marks**

Part A

Answer **any five** *questions.*

Each question carries **4**marks.

1. Define the following
   (i) G set (ii) Isotropy subgroup of $x \in X$ where X is a G set
2. Define normalizer of a subgroup H in G and p-group.
3. State third isomorphism theorem.Describe with an example.
4. Define ascending central series of the group G and give an example.
5. Find the product of the polynomials $f(x) = 4x - 5$ and $g(x) = 2x^2 - 4x + 2$ in $\mathbb{Z}_8[x]$
6. Let $\phi : \mathbb{Z}_{18} \longrightarrow \mathbb{Z}_{12}$ be the homomorphism such that $\phi(1) = 10$. Find $ker\phi$

(5 × 4 = 20 marks)

Part B

Answer **any three** *questions*.

Each question carries **7** *marks*

7. State whether the following statements are ture or false. Justify
   (a) Any abelian group of order 27 is cyclic.
   (b) Any abelian group of order 14 is cyclic.
   (c) Any abelian group of order 21 is cyclic.
   (d) Any abelian group of order 30 is cyclic.
8. Consider a group G. Let $Z(G)$ be its centre for $n \in N$, define $J_n = \{(g_1, g_2, \ldots, g_n) \in Z(G) \times Z(G) \times ... \times Z(G) : g_1 g_2 \ldots g_n = e\}$. As a subset of a direct product groups $G \times G \times ... \times G$ (n times direct product of the group G), is
   (a) not necessarily subgroup

(b) a subgroup but not necessarily a normal subgroup

(c) normal subgroup

(d) isomorphic to the direct product $Z(G) \times Z(G) \times ... \times Z(G)$ ($n-1$ times).

9. Find the isomorphic refinements of the following series

(a) $\{0\} < 60\mathbb{Z} < 20\mathbb{Z} < \mathbb{Z}$and $\{0\} < 245\mathbb{Z} < 49\mathbb{Z} < \mathbb{Z}$

(b) $\{0\} < <5> < \mathbb{Z}_{40}$ and $\{0\} < <4> < \mathbb{Z}_{40}$

10. Give an example of a ring homomorphism $\phi : R \longrightarrow R'$ where R has unity 1 and $\phi(1) \neq 0'$,but $\phi(1)$ is not unity of R'

11. Let I_1 be the ideal generated by x^2+1 and I_2 be the ideal generated by x^3-x^2+x+1 in $\mathbb{Q}[x]$. If $R_1 = \frac{\mathbb{Q}[x]}{I_1}$ and $R_2 = \frac{\mathbb{Q}[x]}{I_2}$, then which of the following options are correct. Give justification for each.

(a) R_1 and R_2 are fields

(b) R_1 is a field but R_2 is not a field

(c) R_1 is an integral domain but R_2 is not an integral domain

(d) R_1 and R_2 are not integral domains.

(3 × 7 = 21 marks)

Part C

Answer **any three** *questions.*

Each question carries **13** *marks.*

12. (a) State and prove Cauchy's theorem **9**

(b) Prove that every group of orderp^2 is abelian **4**

13. (a) Prove that if D is an integral domain and F is a field containing D,then F contains field of quotients of D. **9**

(b) Prove that any two fields of quotients of an integral domain D are isomorphic.**4**

14. (a) State and prove third sylow theorem **9**

(b) Prove that no group of order 48 is simple **4**

15. (a) Let G be a nonzero free abelian group with a finite basis.Then prove that every basis of G is finite and all basis have same number of elements. **8**

(b) Show that a free abelian group contains no nonzero elements of finite order. **5**

16. (a) State and prove Eisenstein criterion. **8**

(b) Prove that anonzero polynomial $f(x) \in F[x]$ of degree n can have at most n zeros in a field F **5**

(3 × 13 = 39 marks)

Kannur University

Model Question Paper

FIRST SEMESTER M.Sc. DEGREE EXAMINATION

Mathematics

MSMAT01C02: Linear algebra

CBCSS - 2023 Admission Onwards

Time: Three Hours **Maximum : 80 Marks**

Part A

Answer **any five** *questions.*

Each question carries **4** marks.

1. Define null space of a linear transformation. Give an example.
2. Let T be a linear operator on R^3 defined by$T(x, y, z) = (x + 2y, x + y + z, 2y + 4z)$. Find the matrix of T with respect to the standard basis
3. Define annihilator of a subspace $S$of a vector space V over the field$F$. Find the annihilator of the zero subspace of a vector space.
4. 4. Show that similar matrices have same characteristic polynomial.
5. (a) Define cyclic vector for a linear operator on a finite-dimensional vector space$V$.

   (b) Define$T$-annihilator of α
6. True or False. Justify. "Every inner product space is a metric space".

(5 × 4 = 20 marks)

Part B

Answer **any three** *questions*.

Each question carries **7** *marks*

7. Let $S : R^3 \to R^4$ and$T : R^4 \to R^3$ be linear transformations such that $T \circ S$ is identity on R^3. Then $S \circ T$ is
   (a) Identity on R^4
   (b) One-One but not onto.
   (c) Onto but not one-one.
   (d) Neither one- one nor onto.

8. Let$A$ be a 2×2 matrix with complex entries which is non-zero and non diagonal. Pick out the cases when$A$is diagonalizable. Justify your answer.

   (a) $A^2 = I$
   (b) $A^2 = 0.$
   (c) All eigen values of A are equal to 2.

9. Give an $n \times n$ matrix B, define $e^B = \sum_{j=0}^{\infty} \frac{B^j}{j!}$ .Let p the characteristic polynomial of B. Then the matrix $e^{(p(B))}$ is.

   (a) $I_{n \times n}$
   (b) $0_{n \times n}$
   (c) $eI_{n \times n}$
   (d) $I_{n \times n}$

   Justify your answer.

10. An $n \times n$ complex matrix satisfies $A^k = I_n$, where$k > 1$positive integer. Suppose$1$is not an eigen value of$A$ Then which are true? Justify your answer.

   (a) A is diagonalizable.
   (b) $A^{k-1} + A^{k-2} + \cdots + A = 0$
   (c) $tr(A^{k-1}) + tr(A^{k-2}) + \cdots + tr(A) = -n$
   (d) $A^{-(k-1)} + A^{(k-2)} + \quad + A^{-1} = I$

11. . Let V be the inner product space consisting of linear polynomials, $p : [0, 1] \to R$ with inner product given by $(p|q) = \int_0^1 p(x)q(x)dx$. An orthonormal basis for$V$ is

   (a) $(1, x)$
   (b) $(1, x\sqrt{3})$
   (c) $(1, (2x - 1)\sqrt{3})$
   (d) $(1, x - 1/2)$

   Justify your answer.

(3 × 7 = 21 marks)

Part C

Answer **any three** *questions.*

Each question carries **13** *marks.*

12. (a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Suppose that V is finite dimensional , then prove that rank (T) + nullity (T) = dim V.

    (b) If A is an m x n matrix with entries in the field F, then prove that row rank (A) = column rank (A)

13. (a) V be a finite-dimensional vector space over the field$F$, and let Wbe a subspace of V. Then prove that $dimW + dimW^o = dimV$

    (b) If W_1 and W_2 are subspaces of a finite- dimensional vector space, then prove that $W_1 = W_2$ if and only if $W_1^o = W_2^o$.

14. (a) State and prove Cayley-Hamilton theorem.

    (b) If W is an invariant subspace for T , then prove that W is invariant under any polynomial in T.

15. (a) .Let V be a finite dimensional vector space over a field F and let T be a linear operator on .Then show that T is triangulable if and only if the minimal polynomial for T is the product of linear polynomials over F.

    (b) Let T be a linear operator on a finite-dimensional space V.If T is diagonalizable and if $c_1, c_2, ., c_k$are the distinct characteristic values of$T$, then prove that there exist linear operators $E_1, E_2, , E_k$on V such that (i)$T = c_1E_1 + c_2E_2 + \cdots + c_kE_k$ (ii)$I = E_1 + E_2 + \cdots + E_k$ (iii) $E_iE_j = 0, i \neq j$

16. State and prove Primary Decomposition Theorem.

(3 × 13 = 39 marks)

Kannur University

Model Question Paper

FIRST SEMESTER M.Sc. DEGREE EXAMINATION

Mathematics

MSMAT01C03: Real Analysis

CBCSS - 2023 Admission Onwards

Time: Three Hours amp; amp;**Maximum : 80 Marks**

Part A

Answer **any five** *questions.*

Each question carries **4** marks.

1. Define a perfect set. Give an example with justification.
2. Define a compact set. Give an example with justification.
3. Give an example of a continuous function which is not uniformly continuous. Justify.
4. Give an example of a real valued function f defined on $[-1, 1]$ such that f has a local maximum at 0.
5. Define Riemann - Stieltjes integral and what is its connection with Riemann integral.
6. (a) State fundamental theorem of calculus.
   (b) Define arc in $\mathbb{R}^k$.

(5 × 4 = 20 marks)

Part B

Answer **any three** *questions.*

Each question carries **7** *marks*

7. Let A be the following subset of $\mathbb{R}^2$: $A = \{(x, y) : (x+1)^2 + y^2 \leq 1\} \cup \{(x, y) : y = x \sin \frac{1}{x}, x > 0\}$. Then which of the following statements is/are true and which of them is/are false ? justify your answer.

   (i) A is connected.
   (ii) A is compact.
   (iii) A is bounded.

8. Suppose f is a uniformly continuous mapping of a metric space X into a metric space Y and prove that $\{f(x_n\}$ is a Cauchy sequence in Y for every Cauchy sequence $\{x_n\}$ in X. Use this result to give an alternate proof of the following fact:
Let E be a dense subset of a metric space X, let f be a uniformly continuous real function defined on E. Prove that f has a continuous extension from E to X.

9. Let $I = [0, 1]$ be the closed unit interval. Suppose f is a continuous mapping of I into I. Prove that $f(x) = x$ for at least one $x \in I$.

10. Suppose f is differentiable on $[a, b]$, $f(a) = 0$, and there is a real number A such that $|f'(x)| \leq A|f(x)|$ on $[a, b]$. Prove that $f(x) = 0$ for all $x \in [a, b]$.

11. Suppose f is twice differentiable function on $(0, \infty)$, f'' is bounded on $(0, \infty)$, and $f(x) \to 0$ as $x \to \infty$. Prove that $f'(x) \to 0$ as $x \to \infty$.

(3 × 7 = 21 marks)

Part C

Answer **any three** *questions.*

Each question carries **13** *marks*

12. (a) Define open sets and closed sets. Prove that a set E is open if and only if its complement is closed.
(b) Define limit point of a set. If p is a limit point of a set E, then every neighborhood of p contains infinitely many points of E.

13. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.
(b) Prove that if f is a continuous mapping of a compact metric space X into a metric space Y, then $f(X)$ is compact.

14. (a) Suppose f is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$. Prove that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$.
(b) Let f be defined on $\mathbb{R}$ by

$$f(x) = \{ \begin{array}{ll} x\sin(\frac{1}{x}), & ;\text{if } x \neq 0, \\ 0, & ;\text{if } x = 0. \end{array}$$

Prove that f is a differentiable at all points x but f' is not a continuous function.

15. (a)Prove that $\underline{\int_a^b} f\, d\alpha \leq \overline{\int_a^b} f\, d\alpha$.
(b) If f is continuous on $[a, b]$, then $f \in \mathcal{R}(\alpha)$ on $[a, b]$.

16. (a) If $a < s < b$, f is bounded on $[a, b]$, f is continuous at s, and $\alpha(x) = I(x - s)$, then

$$\int_a^b f\, d\alpha = f(s).$$

(b) Assume α increases monotonically and $\alpha' \in \mathcal{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then $f \in \mathcal{R}(\alpha)$ if and only if $f\alpha' \in \mathcal{R}$. In that case

$$\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x).$$

(3 × 13 = 39 marks)

Kannur University

Model Question Paper

FIRST SEMESTER M.Sc. DEGREE EXAMINATION

Mathematics

MSMAT01C04: Topology

CBCSS - 2023 Admission Onwards

Time: Three Hours **Maximum : 80 Marks**

Part A

Answer **any five** *questions.*

Each question carries **4** marks.

1. Let X be a set and $\mathcal{T}$ be the collection of all subsets U of X such that $X - U$ is either finite or is all of X. Show that $\mathcal{T}$ is a topology on X.
2. If $X = \{a, b, c\}$, let $\mathcal{T}_1 = \{\phi, X, \{a\}, \{a, b\}\}$ and $\mathcal{T}_2 = \{\phi, X, \{a\}, \{b, c\}\}$. Find the smallest topology containing $\mathcal{T}_1$ and $\mathcal{T}_2$, and the largest topology contained in $\mathcal{T}_1$ and $\mathcal{T}_2$.
3. Define order topology.
4. True or false: Every finite point set in a Hausdorff space is closed. Justify
5. Define metrizable space and give an example.
6. Show that every path connected space is connected.

(5 × 4 = 20 marks)

Part B

Answer **any three** *questions.*

Each question carries **7** *marks*

7. Consider the following topologies on $\mathbb{R}$.
$\mathcal{T}_1 =$ the standard topology,
$\mathcal{T}_2 =$ the $\mathbb{K}-$topology $\mathbb{R}_{\mathbb{K}}$,
$\mathcal{T}_3 =$ the finite complement topology,
$\mathcal{T}_4 =$ the upper limit topology, having all sets $(a, b]$ as basis,
$\mathcal{T}_5 =$ the topology having all sets $(-\infty, a) = \{x|x < a\}$ as basis.
Determine , for each of these topologies , which of the others it contains.

8. Consider the following sets in the dictionary order. Which are linear continua? Explain the reason.
(a) $\mathbb{Z}_+ \times [0,1)$
(b) $[0,1) \times \mathbb{Z}_+$
(c) $[0,1) \times [0,1]$
(d) $[0,1] \times [0,1)$

9. Which of the following are metrics on $\mathbb{R}$. Explain with reasons.
(a) $d(x,y) = min\{x,y\}$
(b) $d(x,y) = |x-y|$
(c) $d(x,y) = |x^2 - y^2|$
(d) $d(x,y) = |x^3 - y^3|$

10. Consider the metrics $d(x,y) = ((x_1 - y_1)^2 + (x_2 - y_2)^2)^{\frac{1}{2}}$ and $\rho(x,y) = max\{|x_1 - y_1|, |x_2 - y_2|\}$ on $\mathbb{R} \times \mathbb{R}$. Prove or disprove: the topologies induced by d and ρ on $\mathbb{R} \times \mathbb{R}$ are the same.

11. Consider the space $S = \{(a,b) | a, b \in \mathbb{Q}\} \subset \mathbb{R}^2$, where $\mathbb{Q}$ is the set of rational numbers. Then which of the following options are correct. Give justification for each.
(a) S is connected in $\mathbb{R}^2$
(b) S^c is connected in $\mathbb{R}^2$, where S^c denotes the complement of S
(c) S is path connected in $\mathbb{R}^2$
(d) S^c is path connected in $\mathbb{R}^2$

(3 × 7 = 21 marks)

Part C

Answer **any three** *questions.*

Each question carries **13** *marks.*

12. (a) Define product topology . If $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y, then show that the collection $\mathcal{D} = \{B \times C \mid B \in \mathcal{B}\ and\ C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$. 7

(b) Define subbasis for a topological space. Show that the collection $\mathcal{S} = \{\pi_1^{-1}(U) \mid U\ open\ inX\} \cup \{\pi_2^{-1}(V) \mid V\ open\ inY\}$ is a subbasis for the product topology on $X \times Y$. 6

13. (a) Let X be an ordered set in the order topology, let Y be a subset of X that is convex in X. Then show that the order topology on Y is the same as the topology Y inherits as a sub space of X. 7

(b) Let Y be a subspace of X. Then show that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y. 6

14. (a) Let X be a space satisfying T_1 axiom and let A be a subset of X. Then show that the point x is a limit point of A if and only if every neighborhood of x contains infinitely many points of A. 5

(b) Let X and Y be topological spaces and let $f : X \to Y$. Then prove the following statements are equivalent.
(1) f is continuous
(2) For every subset A of X, $f(\overline{A}) \subset \overline{f(A)}$
(3) For every closed subset B of Y, the set $f^{-1}(B)$ is closed in X.
(4) For each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$ 8

15. (a) State and prove Uniform limit theorem. 6

(b) Let $p : X \to Y$ be a quotient map and let A be a subspace of X that is saturated with respect to p. Let $q : A \to p(A)$ be the map obtained by restricting p. Prove the following:
(a) If A is either open or closed in X, then q is a qoutient map.
(b) If p is either an open map or a closed map, then q is a quotient map. 7

16. (a) Prove that a finite cartesian product of connected spaces is connected. 8

(b) Prove that a space X is locally connected if and only if for every open set U of X , each component of U is open in X . 5

(3 × 13 = 39 marks)

Kannur University

Model Question Paper

FIRST SEMESTER M.Sc. DEGREE EXAMINATION

Mathematics

MSMAT01C05: Ordinary Differential Equations

CBCSS - 2023 Admission Onwards

Time: Three Hours **Maximum : 80 Marks**

Part A

Answer **any five** *questions.*

Each question carries **4** marks.

1. Express $\sin^{-1} x$ in the form of a power series $\sum a_n x^n$ by solving $y' = (1 - x^2)^{-1/2}$ in two ways.
2. Write Rodrigues' formula. Find the first four Legendre polynomials.
3. Show that
   (a) $\dfrac{d}{dx} J_0(x) = -J_1(x)$
   (b) $\dfrac{d}{dx}[xJ_1(x)] = xJ_0(x)$
4. State the fundamental existence and uniqueness theorem for a linear system of first order differential equations.
5. Find the exact solution of the initial value problem: $y' = 2x(1 + y),\ y(0) = 0$
6. Solve the following initial value problem by Picard's method:

$$\frac{dy}{dx} = z, y(0) = 1$$

$$\frac{dz}{dx} = -y, z(0) = 0$$

(5 × 4 = 20 marks)

Part B

Answer **any three** *questions.*

Each question carries **7** *marks*

7. Consider the differential equation $x(1 - x)y'' + [p - (p + 2)x]y' - py = 0$, where is p is a constant.

(a) If p is not an integer, find the general solution near $x = 0$ in terms of hypergeometric functions.

(b) Write the general solution found in (a) in terms of elementary functions.

(c) What happens when $p = 1$? Find the general solution in this case.

8. Show that the change of dependent variable defined by $y = t''w$ transforms $y'' + [\frac{(1-a-b)-(2-c)t}{t(1-t)}]y' + \frac{ab}{t^2(1-t)}y = 0$ into the hypergeometric equation $t(1-t)w'' + \{(1+a-b) - [a + (1+a-c)+1]t\}w' - a(1+a-c)w = 0$. When $a - b$ is not an integer, find two independent solutions for larger values of x

9. Analyze the number of positive zeros of non-zero solutions of $y'' + (k/x^2)y = 0$, where $k > 0$.

10. Prove the following for a nontrivial solution $y_p(x)$ of Bessel's equation:

    (a) If $0 \leq p < 1$, then every interval of length π contains at least one zero of $y_p(x)$.

    (b) If $p = \frac{1}{2}$, then the distance between successive zeros of $y_p(x)$ is exactly π.

    (c) If $p > \frac{1}{2}$, then every interval of length π contains at most one zero of $y_p(x)$.

11. Let $Q_n(x) = \frac{P_n(x)}{c}$, where c is the leading coefficient of Legendre polynomial $P_n(x)$. Prove that among all polynomials of degree n, $Q_n(x)$ deviates least from zero on the interval $-1 \leq x \leq x$ in the sense of least squares.

(3 × 7 = 21 marks)

Part C

Answer **any three** *questions.*

Each question carries **13** *marks.*

12. (a) Solve by power series method : $y' - y = 0$

    (b) Determine whether $x = 0$ is an ordinary point or regular singular point of the differential equation $2x^2y'' + 7x(x+1)y' + 3y = 0$

13. (a) What is the n^{th} Legendre Polynomial?

    (b) State and prove the orthogonality conditions of sequence of Legendre polynomials on the interval $[-1, 1]$

14. (a) Obtain $J_p(x)$, the Bessel function of the first kind of order p.What is the n^{th} Legendre Polynomial?

    (b) Show that $(n + \frac{1}{2})! = \frac{(2n+1)!}{2^{2n+1}n!}\sqrt{\pi}$

15. (a) Find the general solution of the system:

$$\frac{dx}{dt} = 3x - 4y$$

$$\frac{dy}{dt} = x - y$$

(b) Let W(t) be the Wronskian of the two solutions $x = x_1(t); y = y_1(t)$ and $x = x_2(t); y = y_2(t)$ of the system

$$\frac{dx}{dt} = a_1(t)x + b_1(t)y$$

$$\frac{dy}{dt} = a_2(t)x + b_2(t)y$$

Show that $w(t)$ is either identically zero or nowhere zero on the specified interval.

16. Let $f(x, y)$ be a continuous function that satisfies Lipschitz condition on the infinite strip $a \leq x \leq b;\ -\infty < y < \infty$. If (x_0, y_0) is any point of the strip, then show that $y' = f(x, y);\ y(x_0) = y_0$ has a unique solution on the interval $a \leq x \leq b$.

(3 × 13 = 39 marks)